Stacking seismic data using local correlation[a]

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Guochang Liu[*†], *Sergey Fomel*[†], *Long Jin*[‡], *Xiaohong Chen*[*]

ABSTRACT

Stacking plays an important role in improving signal-to-noise ratio and imaging quality of seismic data. However, for low-fold-coverage seismic profiles, the result of conventional stacking is not always satisfactory. To address this problem, we have developed a method of stacking in which we use local correlation as a weight for stacking common-midpoint gathers after NMO processing or common-image-point gathers after prestack migration. Application of the method to synthetic and field data showed that stacking using local correlation can be more effective in suppressing random noise and artifacts than other stacking methods.

INTRODUCTION

Stacking as one of the three crucial techniques (deconvolution, stacking, and migration) plays an important role in improving signal-to-noise ratio (S/R) in seismic data processing (Yilmaz, 2001). Conventional stacking, which is performed by averaging an NMO-corrected data set or migrated data set, is optimal only when noise components in all traces are uncorrelated, normally distributed, stationary, and of equal magnitude (Mayne, 1962; Neelamani et al., 2006). Therefore, different stacking technologies have been proposed, along with improvements in optimizing weights of seismic traces.

Robinson (1970) proposes using an S/N-based weighted stack to further minimize noise. Using cross-correlation of seismic traces and normalized cross-correlation processing, Chang et al. (1996) proposes preserved frequency stacking. Schoenberger (1996) proposes optimum weighted stack for multiple suppression, with weight determined by solving a set of optimization equations. Neelamani et al. (2006) propose a stack-and-denoise method called SAD, which exploits the structure of seismic signals to obtain an enhanced stack. Zhang and Xu (2006) present a high-order correlative weighted stacking technique on the basis of wavelet transformation and high-order statistics. By estimating the probability distribution of noise, Trickett (2007) applies a maximum-likelihood estimator to stacking. To eliminate artifacts in angle-domain common-image gathers (CIGs) caused by sparsely sampled wavefields, Tang (2007) presents a selective stacking approach that applies local smoothing of envelope function to achieve the weighting function. Rashed (2008) proposes smart stacking, which is based on optimizing seismic amplitudes of the stacked signal by excluding harmful samples from the stack and applying larger weight to the central part of the sample population.

The global correlation coefficient can measure the similarity of two signals, but it is not a local attribute. Not only does the sliding-window global-correlation approach need many parameters to be specified, but this approach cannot smoothly characterize thin layers well. Fomel (2007b) uses shaping regularization, which controls locality and smoothness to define local correlation (Fomel, 2007a). Local correlation is applied to multicomponent seismic image registration (Fomel et al., 2005; Fomel, 2007a) and time-lapse image registration (Fomel and Jin, 2007).

In this paper,we present a new stacking method using local correlation. This method applies time-dependent smooth weights (which are taken as local correlation coefficients between reference traces and prestack traces), stacks the common-midpoint (CMP) gather, and effectively discards parts of the data that least contribute to stacked reflection signals. Using synthetic and field data examples, we show that, compared with other stacking methods, this method can greatly improve the S/N and suppress artifacts.

METHODOLOGY

Review of local correlation

The global uncentered correlation coefficient between two discrete signals $\mathbf{a}_i$ and $\mathbf{b}_i$ can be defined as the functional

$$\gamma = \frac{\sum_{i=1}^{N} \mathbf{a}_i \mathbf{b}_i}{\sqrt{\sum_{i=1}^{N} \mathbf{a}_i^2 \sum_{i=1}^{N} \mathbf{b}_i^2}} , \tag{1}$$

where N is the length of a signal. The global correlation in equation 1 supplies only one number for the whole signal. For measuring the similarity between two signals locally, one can define the sliding-window correlation coefficient

$$\gamma_w(t) = \frac{\sum_{i=t-w/2}^{t+w/2} \mathbf{a}_i \mathbf{b}_i}{\sqrt{\sum_{i=t-w/2}^{t+w/2} \mathbf{a}_i^2 \sum_{i=t-w/2}^{t+w/2} \mathbf{b}_i^2}} , \tag{2}$$

where w is window length.

Fomel (2007a) proposes the local correlation attribute that identifies local changes in signal similarity in a more elegant way. In a linear algebra notation, the correlation coefficient in equation 1 can be represented as a product of two least-squares inverses γ_1 and γ_2:

$$\gamma^2 = \gamma_1\gamma_2 \ , \tag{3}$$

$$\gamma_1 = \arg\min_{\gamma_1} \parallel \mathbf{b} - \gamma_1\mathbf{a} \parallel^2 = (\mathbf{a}^T\mathbf{a})^{-1}(\mathbf{a}^T\mathbf{b}) \ , \tag{4}$$

$$\gamma_2 = \arg\min_{\gamma_2} \parallel \mathbf{a} - \gamma_1\mathbf{b} \parallel^2 = (\mathbf{b}^T\mathbf{b})^{-1}(\mathbf{b}^T\mathbf{a}) \ , \tag{5}$$

where $\mathbf{a}$ and $\mathbf{b}$ are vector notions for $\mathbf{a}_i$ and $\mathbf{b}_i$. Let $\mathbf{A}$ and $\mathbf{B}$ be two diagonal operators composed of the elements of a and b. Localizing equations 4 and 5 amounts to adding regularization to inversion. Using shaping regularization (Fomel, 2007b), scalars γ_1 and γ_2 turn into vectors $\mathbf{c}_1$ and $\mathbf{c}_2$, defined as

$$\mathbf{c}_1 = [\lambda^2\mathbf{I} + \mathbf{S}(\mathbf{A}^T\mathbf{A} - \lambda^2\mathbf{I})]^{-1}\mathbf{S}\mathbf{A}^T\mathbf{b} \ , \tag{6}$$

$$\mathbf{c}_2 = [\lambda^2\mathbf{I} + \mathbf{S}(\mathbf{B}^T\mathbf{B} - \lambda^2\mathbf{I})]^{-1}\mathbf{S}\mathbf{B}^T\mathbf{a} \ , \tag{7}$$

where λ scaling controls relative scaling of operators $\mathbf{A}$ and $\mathbf{B}$ and where $\mathbf{S}$ is a shaping operator such as Gaussian smoothing with an adjustable radius. The component-wise product of vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ defines the local correlation measure. Local correlation is a measure of the similarity between two signals. An iterative, conjugate-gradient inversion for computing the inverse operators can be applied in equations 6 and 7. Interestingly, the output of the first iteration is equivalent to the algorithm of fast local cross-correlation proposed by Hale (2006).

Stacking using local correlation

The problem of combining a collection of seismic traces into a single trace is commonly referred to as stacking in seismic data processing. This process is used to attenuate random noise and simultaneously amplify the coherent signal in the gather. Typically, the desired stacked trace is estimated by averaging traces from the CMP gather (Mayne, 1962):

$$\bar{a}_j(t) = \frac{1}{N}\sum_{i=1}^{N} a_{i,j}(t), \qquad j = (1, 2, 3, \ldots, M) \ , \tag{8}$$

where N is the fold of the stack and $a_{i,j}(t)$ is the sample value on trace i from the jth CMP gather at two-way time t. Such a technique provides the optimal unbiased estimate of $\bar{a}_j(t)$. Robinson (1970) proposes weighted stacking of seismic data:

$$\bar{a}_j(t) = \frac{1}{\sum_{i=1}^{N} w_{i,j}}\sum_{i=1}^{N} w_{i,j}\cdot a_{i,j}(t), \qquad j = (1, 2, 3, \ldots, M) \ , \tag{9}$$

where $w_{i,j}$ denotes the weight of the ith trace from the jth CMP gather, which is determined by noise variances $w_{i,j} = 1/\sigma_{i,j}^2$. However, it is difficult to estimate noise variances reliably in practice. Neelamani et al. (2006) use an iterative algorithm called "leave me out" (LMO) to estimate noise variances from data. The desired signal is assumed to be flat with constant amplitude across all the traces within a gather in the LMO method.

For using time-dependent smooth weights in the stacking process, we choose the local correlation coefficient from the previous section as weights to stack seismic data. We find that local correlation better characterizes local similarity between prestack and reference traces than the sliding-window method.

Consider the two noisy signals with Gaussian random noise but different noise levels in Figure 1c and 1d. The signals are derived from adding noise on convolution of the Ricker wavelet (Figure 1a) with synthetic reflectivity (Figure 1b). The distribution of noise in (Figure 1c) is $N(\mu,\sigma) = N(0,10^{-6})$, where μ and σ are mean and variance of noise, respectively. The distribution of noise in (Figure 1d) is $N(\mu,\sigma) = N(0,0.07)$. The sliding-window correlation and local correlation between Figure 1c and Figure 1d are shown in Figure 1e and Figure 1f, respectively. Note that local correlation coefficients (Figure 1f) are smooth and better distinguish the thin layer, represented by the first two reflectivities in Figure 1b. In application to stacking, the prestack trace is analogous to Figure 1d with larger variance noise, and the reference trace is analogous to Figure 1c with smaller variance noise.

To implement seismic data stacking using local correlation, we first apply the equal-weight stack to the NMO-corrected CMP gather to obtain the reference trace. Then we compute the local correlation coefficients between the reference trace and the NMO-corrected CMP gather and apply soft thresholding (Donoho, 1995) to all local correlation coefficients. Finally, we apply the weighted stack to the CMP gather using local correlation to get the final stacked trace. Mathematically, stacking using the local correlation approach modifies equation 9 to

$$\bar{a}_j(t) = \frac{1}{K_j H_j(t)} \sum_{j=1}^{N} w_{i,j} \cdot a_{i,j}(t), \qquad j = (1,2,3,\ldots,M)\ , \tag{10}$$

$$w_{i,j}(t) = \begin{cases} \eta_{i,j}(t) - \varepsilon, & \eta_{i,j} > \varepsilon \\ 0, & \eta_{i,j} \leq \varepsilon \end{cases}\ , \tag{11}$$

where ε is the threshold value, $K_j = \sum_{t=0}^{t}\sum_{i=0}^{N} w_{i,j}(t)$ is the sum of weights of the jth CMP gather, $H_j(t)$ is the number of samples with $w_{i,j} \cdot a_{i,j}(t) \neq 0$ for a given two-way time, and $\eta_{i,j}(t)$ is the local correlation between the ith prestack trace from jth gather and the reference trace. The local correlation $\eta_{i,j}(t)$ can be computed using equations 6 and 7. The reference trace is derived from averaging all the traces of one CMP gather. Here we assume that the equal-weight stacked trace is close to the desired trace. Because the weights are a function of two-way traveltime and offset, recovery scalar K_j has the same value for the same CMP gather. Meanwhile, the samples with $w_{i,j}(t) \cdot a_{i,j} \neq 0$ at a given two-way time are assumed to be full noise or null value such as muting parts; we therefore use $H_j(t)$ to scale the stacked trace.

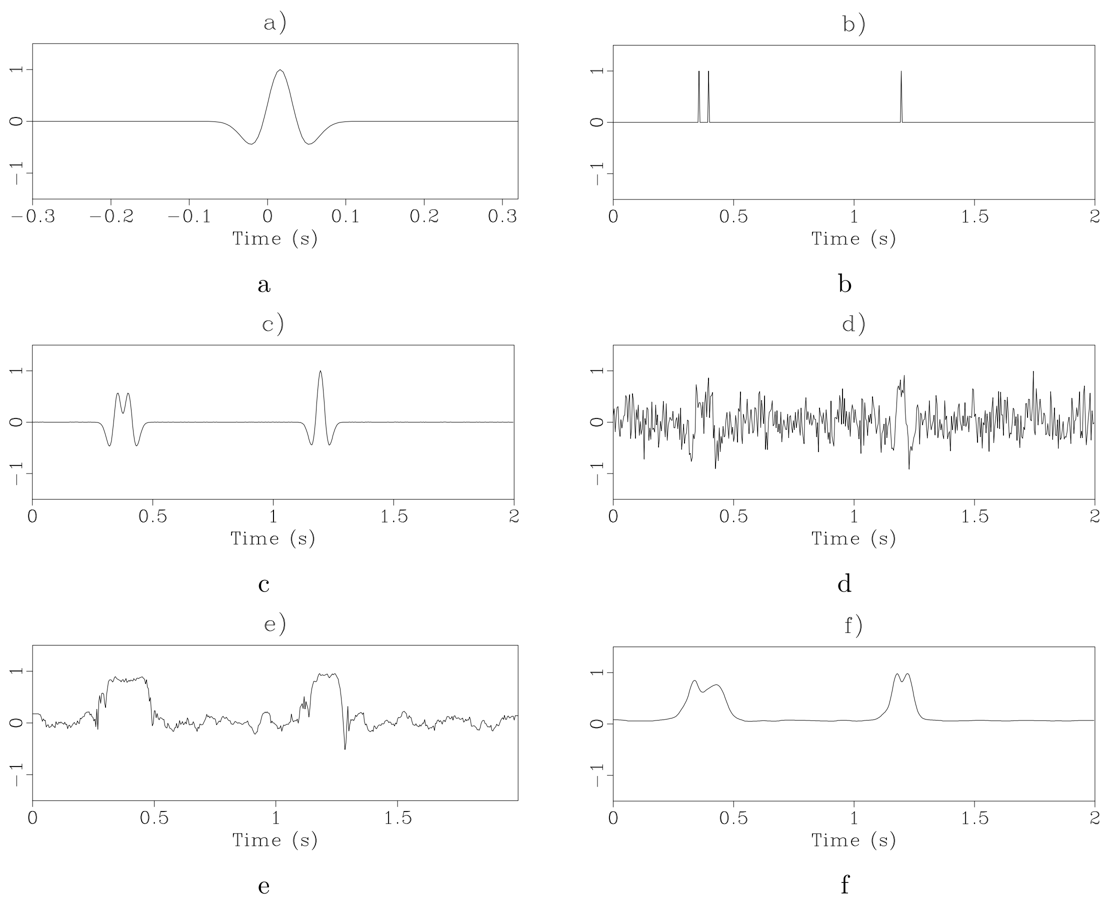

Figure 1: (a) Zero-phase Ricker wavelet. (b) Reflection coefficient. (c) Noisy signal with $N(\mu,\sigma) = N(0, 10^{-6})$. (d) Noisy signal with $N(\mu,\sigma) = N(0, 0.07)$. (e) Sliding-window correlation. (f) Local correlation.

Changes occurring between equation 9 and equations 10 and 11 result from time-dependent smooth weights for the stack and application of thresholding to the weights. All local correlation coefficients below a specified threshold are discarded, and the rest, with values above the threshold, are included. We thus stack only those parts of prestack data whose similarity to the reference trace is comparatively large. Equations 10 and 11 implicitly estimate the noise variance by analyzing local cross-correlations between prestack trace and the reference trace. This operation can be understood as a nonlinear filter that enhances the coherency of events. We perform this operation for all gathers using this method to improve the stack profile.

When applied after angle-domain migration, normalization provided by soft thresholding is analogous to true-amplitude illumination compensation from the so-called Beylkin determinant (Albertin et al., 1999; Audebert and Froidevaux, 2005). Local correlation normalizes the image by the number of strongly illuminated angles in angle-domain CIGs.

In the following, we discuss the distinctions between seismic stacking using local correlation and other methods. Our method creates time-dependent smooth weights without depending on sliding windows, as compared to other weighted stacking methods such as statistically optimal stacking (Robinson, 1970; Neelamani et al., 2006) and the semblance method (Yilmaz, 2001). In contrast to smart stacking, proposed by (Rashed, 2008) and based on sign difference between sample point and the alpha-trimmed mean to remove frequency distortions, our method applies waveform similarity between prestack trace and mean to make the stacking selective.

EXAMPLES

To illustrate the proposed method using synthetic and field data, we apply our approach to three examples. The first example is a simple case involving a fivefold prestack gather (Figure 2a) with a timeshifted-upward trace, which might be distortion by poor static correction. The peak of the signal in this gather is one.We add Gaussian random noise with distribution $N(\mu, \sigma) = N(0, 0.01)$ on the five traces. The result of an equal-weight stack is shown in Figure 2c. The upside wing in Figure 2c is distorted because of the first time-shift trace. Then we use three weighted stacking methods to stack the five traces.

Figure 2d and Figure 2e illustrates results of smart stacking (Rashed, 2008) and LMO-based weighted stacking in which the weights are computed by the LMO method (Robinson, 1970; Neelamani et al., 2006). Figure 2f shows the result of stacking using local correlation with weights (Figure 2b) determined by the similarity between the prestack trace (Figure 2a) and the reference (Figure 2c). Because the waveform in the first trace in Figure 2a is most likely noise or artifact, it is reasonable that the weight in the stack procedure is lower. Use of local correlation as weights of prestack traces lets us select those portions, which are more similar to the reference trace to contribute to the stack.

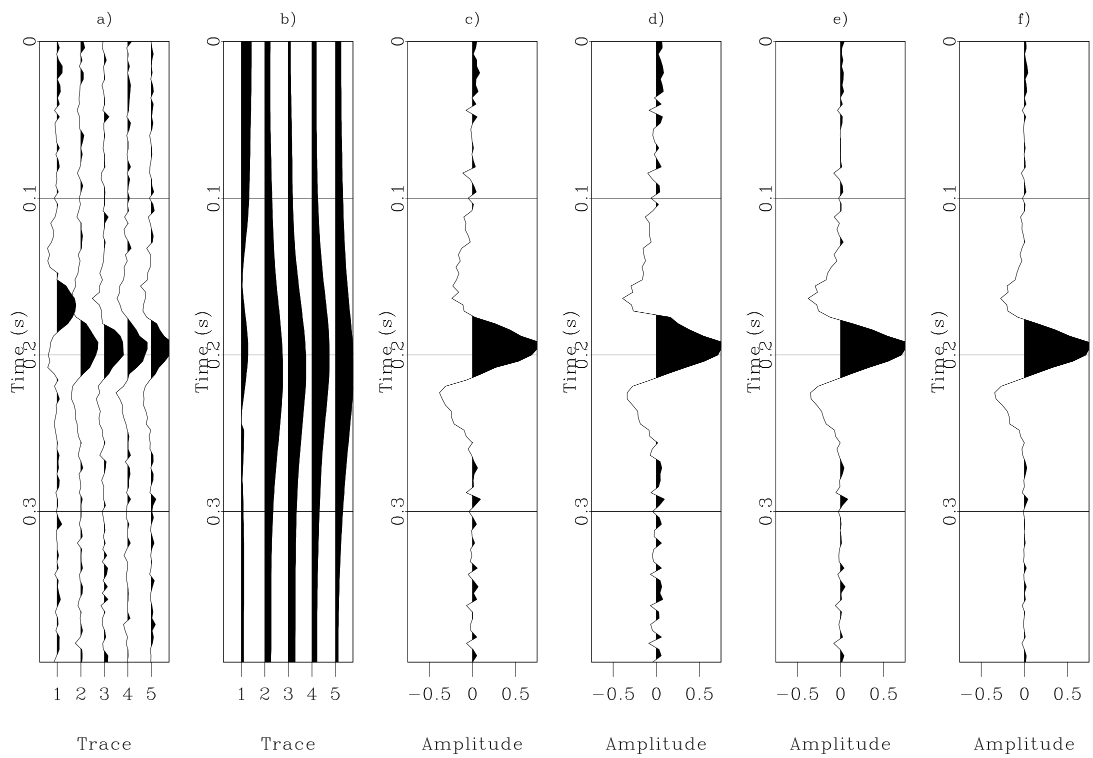

Figure 2: Simple stacking test with fivefold gather. (a) Prestack gather. (b)Weights used in local-correlation weighted stacking. (c) Conventional equal-weight stacking method (S/N=8.4). (d) Smart stacking method (S/N=9.2). (d) LMO-based weighted method (S/N=10.2). (f) Local-correlation weighted stacking (S/N=13.5).

Comparing the three methods, one can find that smart stacking and LMO-based weighted stacking can remove upside wing distortion cleanly, but stacking using local correlation removes more random noise than the other two methods and meanwhile corrects upside wing distortion.

To judge the effect of denoising quantitatively between different methods, we apply equal-weight stacking on the last four traces without any noise to get the exact desired stacked trace $d_j(t)$, which can be regarded as a signal trace. The S/N of the jth CMP can therefore be estimated as

$$\mathrm{S/N}_j = 10\log_{10}\left(\frac{\sum\limits_t [d_j^2(t)]}{\sum\limits_t [d_j(t) - \bar{a}_j(t)]^2}\right) , \tag{12}$$

where $\bar{a}_j(t)$ is the stacked trace from different stacking methods. In the first simple example Figure 2, the S/N of equal-weight stacking is 8.4 dB and the other three weighted methods are, respectively, 9.2, 10.2, and 13.5 dB. Stacking using local correlation can improve S/N greatly.

The second example is a 2D synthetic model that includes four reflectors. Synthetic data are generated with Kirchhoff modeling. The peak of the data set is one and Gaussian random noise with distribution $N(\mu, \sigma) = N(0, 0.05)$ is added. We show the results of stacking one CMP gather (Figure 3a) by three methods in Figure 3c-e. Compared to other methods, our method is the most effective in denoising.

The stacked profile of all CMP gathers is shown in Figure 4. We use equation 12 to compute the S/N of the stacked profile (Figure 4). The S/Ns of three methods are 7.1, 9.6, 10.9 dB, respectively. Noise is attenuated more effectively in the stacking result using local correlation (Figure 4c).

The third example involves a historic 2D line from the Gulf of Mexico (Claerbout, 2005). The stacked sections, using three different methods, are shown in Figure 5. Figure 6 shows the local correlation cube between prestack and reference traces. Similar cubes have been used in multicomponent seismic image registration (Fomel et al., 2005; Fomel, 2007a) and time-lapse image registration (Fomel and Jin, 2007). For synthetic data, the exact desired stacked section can be calculated by stacking prestack traces without any noise. But for field data, the S/N is difficult to estimate using equation 12. We therefore use singular value decomposition (SVD) Andrews and Patterson (1976) to evaluate different stacking methods. The SVD of stacked section matrix gives

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^T . \tag{13}$$

The diagonal elements σ_r of Σ are the singular values of $\mathbf{M}$. The S/N can be estimated as (Freire and Ulrych, 1988; Peterson and DeGroat, 1988; Grion and Mazzotti,

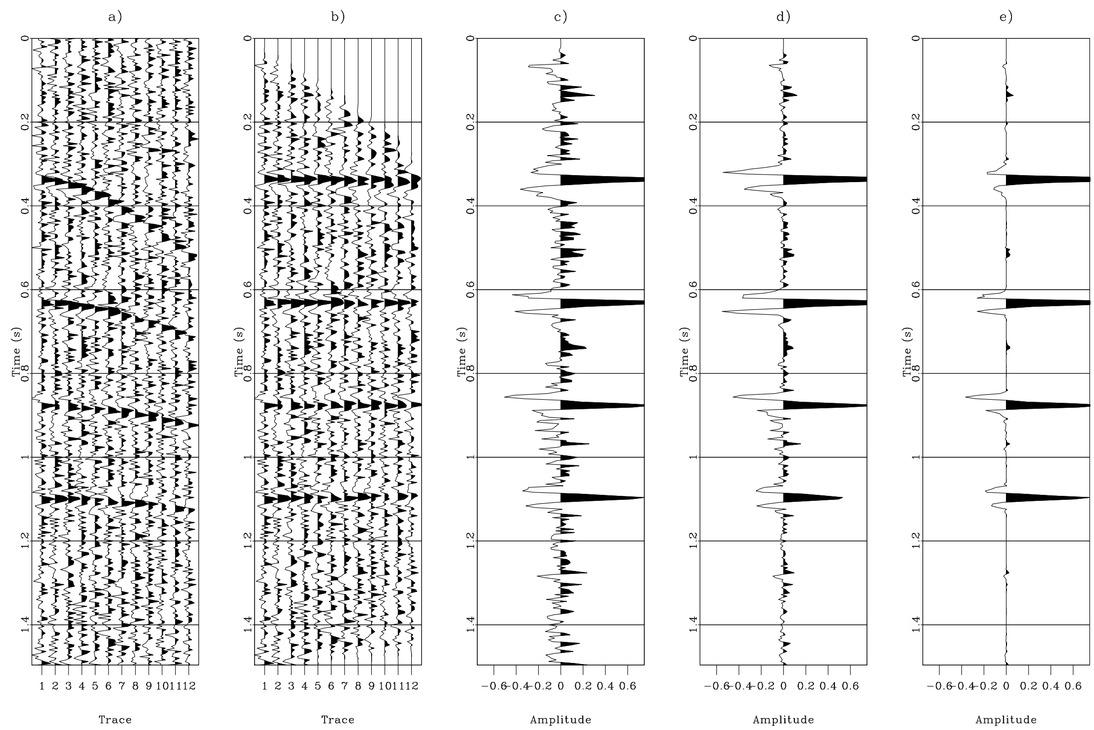

Figure 3: (a) One CMP gather from synthetic data set. (b) NMO-corrected gather. (c) Result of conventional equal-weight stacking. (d) Result of LMO-based weighted stacking. (e) Result of local-correlation weighted stacking.

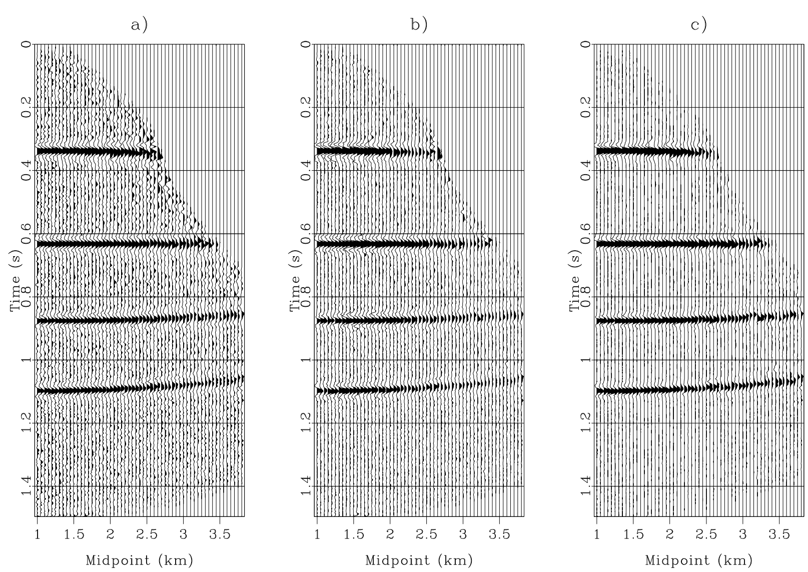

Figure 4: Comparison among three stacking methods including all synthetic CMP gathers. (a) Conventional equal-weight stacking (S/N=7.1). (b) LMO-based weighted stacking (S/N=9.6). (c) Local-correlation weighted stacking (S/N=10.9).

1998)

$$\text{S/N} = 10\log_{10}\left(\frac{\sigma_1^2 - \frac{1}{R-1}\sum_{r=2}^{R}\sigma_r^2}{\frac{1}{R-1}\sum_{r=2}^{R}\sigma_r^2}\right), \tag{14}$$

where R is the number of all singular values. The S/Ns of stacked sections resulting from three stacking methods are, respectively, 27.4, 29.2, and 33.9 dB. Comparing Figure 5a-c, we can find also that random noise is attenuated and coherent reflections are enhanced better using local correlation (e.g., 0.5–1.5-s range).

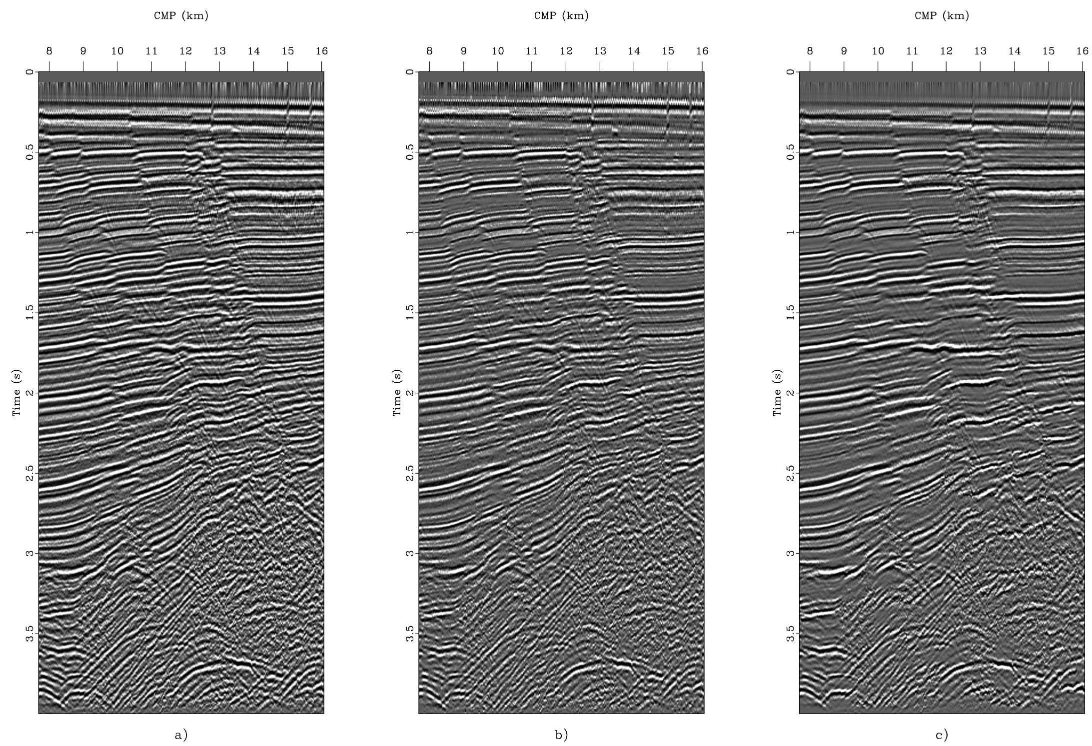

Figure 5: Results of (a) conventional equal-weight stacking (S/N=27.4), (b) LMO-based weighted stacking (S/N=29.2) and (c) local-correlation weighted stacking (S/N=33.9).

CONCLUSION

We have developed a new method of stacking NMO-corrected or migrated seismic data using local correlation. We substitute local correlation for the weight value in statistically weighted stacking and then use soft thresholding to make the stacking selective. Because weights are derived from the input data, our method can be regarded as a nonlinear filter. Synthetic and field data examples confirm that our approach

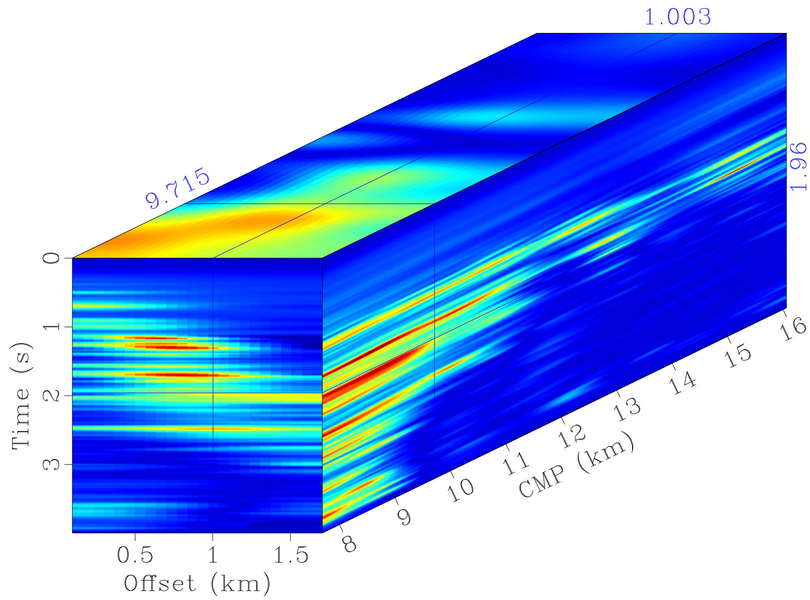

Figure 6: Local correlation cube of the field-data example.

can be significantly more effective than other weighted stacking methods in improving S/N and suppressing distortions resulting from prestack processing. Seismic stacking using local correlation can give a poor result if the quality of the reference trace is very poor. Because the coherency enhancement from local correlation is not based on physics, one should use this approach with caution when aiming to preserve physically meaningful amplitudes.

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